Q $W_t$ is a Standard Brownian Motion. What is the probability that $W(1)>0$ and $W(2)<0$

$$P(W(1)>0 \text{ and } W(2)<0)$$

$$= \int_{-\infty}^{\infty} P(W(1)>0, W(2)<0 \mid W(1)=x)\,\phi(x)\,dx$$

by LFTP

$$= \int_{0}^{\infty} P(W(2)<0 \mid W(1)=x)\,\phi(x)\,dx$$

$$= \int_{0}^{\infty} P(W(2)-W(1)<-x)\,\phi(x)\,dx$$

$$= \int_{0}^{\infty} \left[\int_{-\infty}^{-x} \phi(y)\,dy\right] \phi(x)\,dx$$

$$= \int_{0}^{\infty}\int_{-\infty}^{-x} \frac{1}{2\pi} \exp\left(-\frac{x^2+y^2}{2}\right) dx\,dy = \frac{1}{8}$$

Method 2:

$A = \{W(1)>0\}$ $\quad B = \{W(2)-W(1)<0\}$

$C = \{|W(2)-W(1)| > |W(1)|\}$

$$P(W(1)>0, W(2)<0) = P(A \text{ and } B \text{ and } C)$$

$$= P(A)\cdot P(B)\cdot P(C)$$

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$

$$= \frac{1}{8}$$

note that if $X_1, X_2$ are continuous and i.i.d., then

$P(X_1>X_2) = P(X_1<X_2)$ and $P(X_1=X_2)=0$

$P(|X_1|>|X_2|) = P(|X_1|<|X_2|)$ and $P(|X_1|=|X_2|)=0$